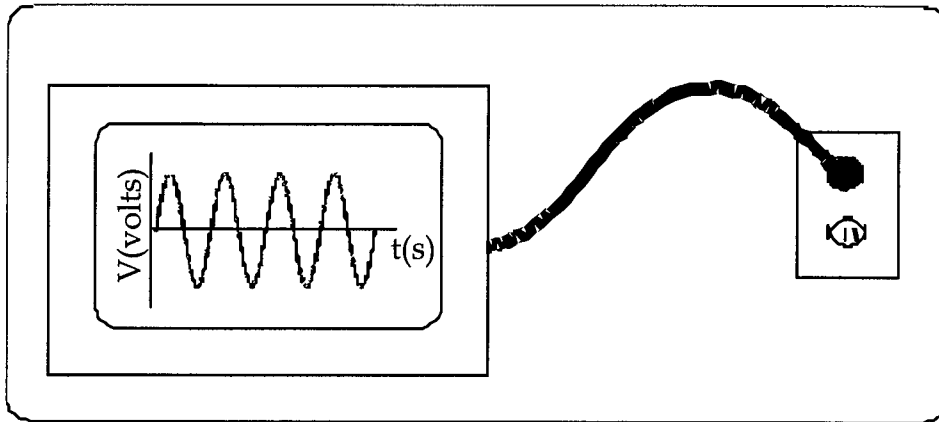


Name _____ Date _____ Partners _____

INTRODUCTION TO AC CURRENTS AND VOLTAGES

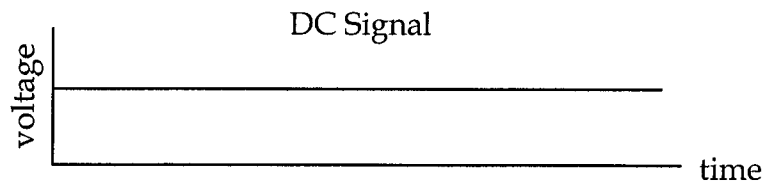


OBJECTIVES

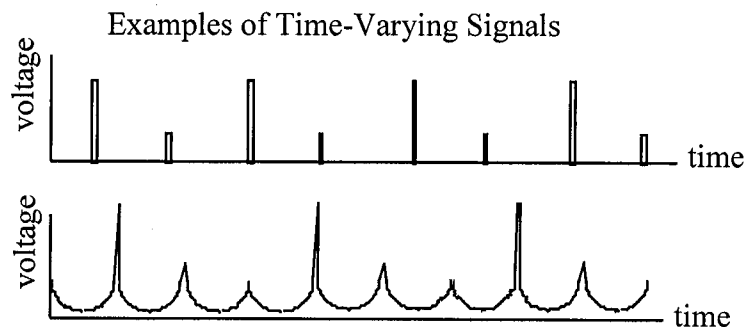
- To learn the meanings of *peak voltage* and *frequency* for AC signals.
- To observe the behavior of resistors in AC circuits.
- To observe the behaviors of capacitors and inductors in AC circuits.
- To understand the meanings of *phase*, *amplitude*, *reactance*, and *impedance* in AC circuits.

OVERVIEW

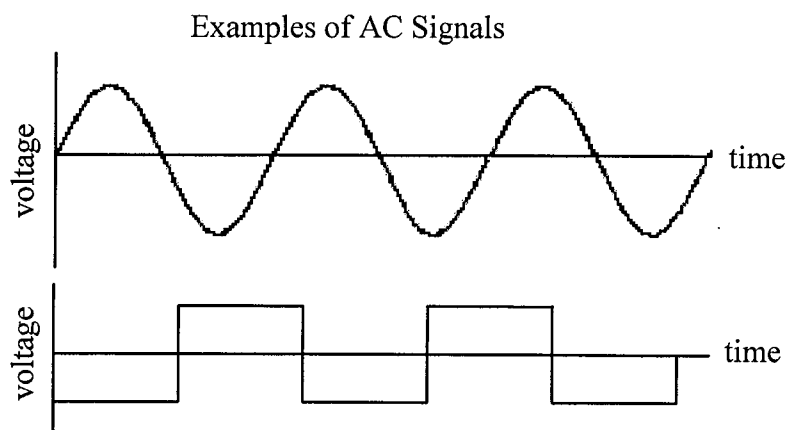
Until now, you have investigated electric circuits in which a battery provided an input voltage that was effectively constant in time. This is called a *DC* or *Direct Current* signal. (A steady voltage applied to a circuit eventually results in a steady current. Steady voltages are usually called *DC* voltages.)



Signals that change over time exist all around you, and many of these signals change in a regular manner. For example, the electrical signals produced by your beating heart change continuously in time.



There is a special class of time-varying signals. These signals can be used to drive current in one direction in a circuit, then in the other direction, then back in the original direction, and so on. They are referred to as *AC* or *Alternating Current* signals.



It can be shown that any periodic AC signal can be represented as a sum of weighted sines and cosines (known as a *Fourier* series). It can also be shown that the response of a circuit containing resistors, capacitors, and inductors (an "*RLC*" circuit) to such a signal is simply the sum of the responses of the circuit to each sine and cosine term with the same weights. We further note that a cosine is just a sine that is shifted back in time by $\frac{1}{4}$ cycle (a phase shift of -90° or $-\pi/2$ radians). So, to analyze an *RLC* circuit we need only find the response of the circuit to an input sine wave of arbitrary frequency.

Let us suppose that we have found a way to generate an input current of the form:

$$I(t) = I_{\max} \sin(\omega t) \quad (1)$$

We have already seen that the voltage across a resistor is then given by:

$$V_R(t) = RI_{\max} \sin(\omega t) \quad (2a)$$

Without proof¹ we will state that the voltage across a capacitor is given by:

$$V_C(t) = -\frac{I_{\max}}{\omega C} \cos(\omega t) = \frac{I_{\max}}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right) \quad (2b)$$

and the voltage across an inductor is given by:

$$V_L(t) = \omega L I_{\max} \cos(\omega t) = \omega L I_{\max} \sin\left(\omega t + \frac{\pi}{2}\right) \quad (2c)$$

These can all be written in the form (a generalized Ohm's law):

$$V(t) = I_{\max} Z \sin(\omega t + \varphi) \quad (3)$$

Arbitrary combinations of resistors, capacitors and inductors will have voltage responses of this form. Z is called the *impedance* and φ is called the *phase shift*. The maximum voltage will be given by

$$V_{\max} = I_{\max} Z \quad (4)$$

For a series combination of a resistor, a capacitor and an inductor,

$$Z_{\text{series}} = \sqrt{R^2 + (X_L - X_C)^2} \quad (5a)$$

and

$$\tan(\varphi_{\text{series}}) = \frac{X_L - X_C}{R} \quad (5b)$$

where

$$X_C \equiv \frac{1}{\omega C} \quad (5c)$$

and

$$X_L \equiv \omega L \quad (5d)$$

X_C is called the *capacitive reactance* and X_L is called the *inductive reactance*. If there is only a capacitor or only an inductor, the impedance is simply the corresponding reactance.

In Investigation 1, you will explore how a time-varying signal affects a circuit with a resistor. In Investigation 2, you will explore how inductors and capacitors influence the current and voltage in various parts in an AC circuit.

¹ You can verify these equations by plugging them into $V_C = q/C$ (or $dV_C/dt = I/C$) and $V_L = L \frac{dI}{dt}$.

INVESTIGATION 1: AC SIGNALS AND RESISTANCE

In this investigation, you will consider the behavior of resistors in a circuit driven by AC signals of various frequencies.

You will need the following materials:

- current probe and voltage probe
- 100 Ω resistor
- multimeter
- alligator clip leads
- internal signal generator of *Data Studio*

Activity 1-1: Resistors and Time-Varying (AC) Signals.

Consider the circuit in Figure 9-1 with a signal generator and resistor.

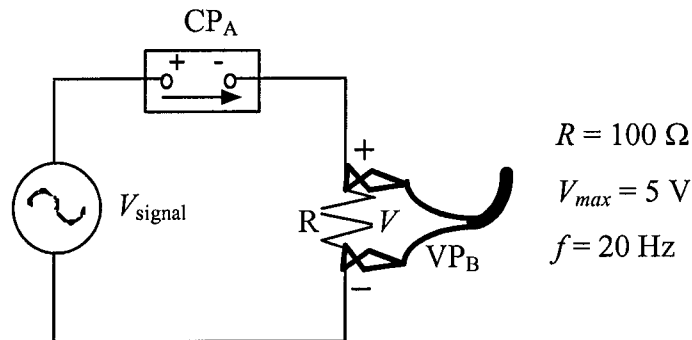
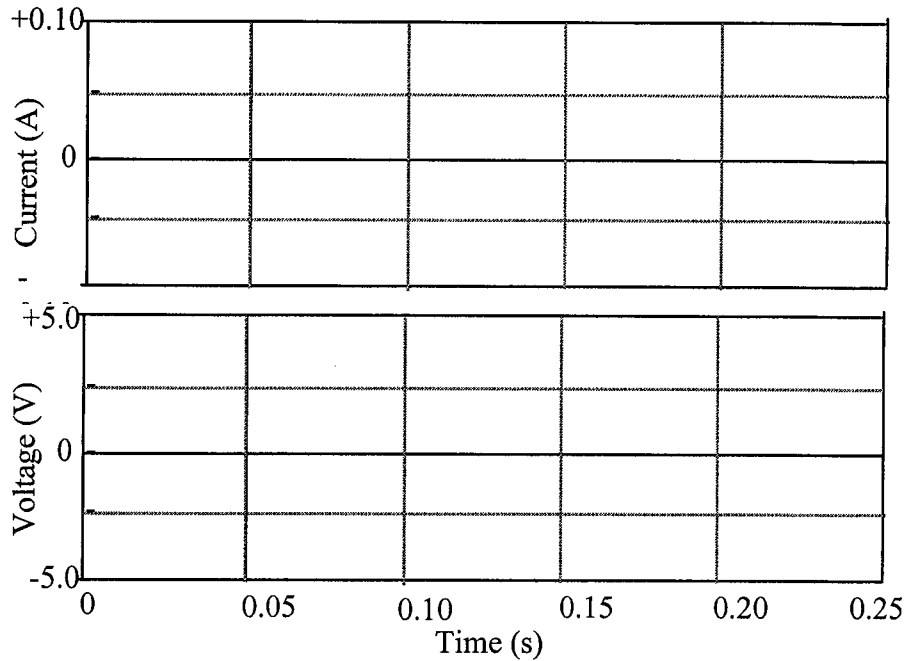


Figure 9-1: Resistor circuit with AC input signal.

Question 1-1: What is the relationship between the input signal, V_{signal} , and the voltage measured by the voltage probe, V ? (**Hint:** remember that CP_A has a very small resistance compared to R .)

Prediction 1-1: On the axes that follow, sketch, with *dotted* lines, your *quantitative* prediction for the input signal, V_{signal} vs. t and the current through the resistor, I vs. t . (**Hint:** consider Ohm's Law). You know the signal voltage and frequency from Figure 9-1, so try to put in correct values.



Test your predictions.

1. Open the experiment file called **Resistor with AC**.
2. Measure the actual resistance of R: _____
3. Connect the circuit in Figure 9-1. We are using the internal signal generator of *Data Studio* interface. The controls should appear on the computer screen.
4. Set the signal generator to 20 Hz and 5 volts amplitude (+5 V maximum and -5 V minimum).
5. **Begin** graphing. When you have a good graph of the signal, **stop** graphing. Expand the graph to look at the same range (0.25 s) as above.
6. **Print** one set of graphs for your group report. Do not erase data.
7. On the graph of voltage vs. time, identify and label a time or two when the current through the resistor is maximum. Depending on the way you hooked up the voltage probe across the resistor, you may have current and voltage in or out of phase. If out of phase, you may want to switch the voltage probe and repeat.
8. On your graph of current vs. time, identify and label a time or two when the voltage across the resistor is maximum.

Question 1-2: Does a voltage maximum occur at the same time as a current maximum, or does one maximum (current or voltage) occur before the other? Explain.

Question 1-3: Use the **Smart Tool** to find the period (time from one peak to the next), T , of the voltage.

T : _____ s

8. Use your graph to complete Column I in **Table 9-1**. To obtain information from the graph, you can use the **Smart Tool** or you can select several cycles by highlighting them, and then use the **statistics feature** to find the maximum values for the voltage and current.
9. Set the frequency of the signal generator to 30 Hz. Check that the amplitude is still 5 V. **Graph** I and V as before. Use the **analysis feature** to complete Column II in **Table 9-1**.
10. Set the frequency of the signal generator to 40 Hz. Check that the amplitude is still 5 V. **Graph** I and V as before, and complete Column III in **Table 9-1**.

Table 9-1

Column I	Column II	Column III
$f = 20\text{Hz}$	$f = 30\text{Hz}$	$f = 40\text{Hz}$
At maximum voltage, current is (circle one): maximum, minimum, zero and increasing, zero and decreasing, nonzero and increasing, nonzero and decreasing, other	At maximum voltage, current is (circle one): maximum, minimum, zero and increasing, zero and decreasing, nonzero and increasing, nonzero and decreasing, other	At maximum voltage, current is (circle one): maximum, minimum, zero and increasing, zero and decreasing, nonzero and increasing, nonzero and decreasing, other
max. voltage (V_{\max}) = _____	max. voltage (V_{\max}) = _____	max. voltage (V_{\max}) = _____
max. current (I_{\max}) = _____	max. current (I_{\max}) = _____	max. current (I_{\max}) = _____
$R = V_{\max}/I_{\max} =$ _____	$R = V_{\max}/I_{\max} =$ _____	$R = V_{\max}/I_{\max} =$ _____

Question 1-4: Based on the calculations in **Table 9-1**, what can you say about the resistance of R at different frequencies (does its value

appear to increase, decrease, or stay the same as frequency increases)? Explain your answer.

Question 1-5: When the input signal is 30 Hz or 40 Hz, does a maximum positive current through R occur before, after, or at the same time as the maximum positive voltage across R ?

Note: Do *not* disconnect this circuit as you will be using a very similar one in Investigation 2.

Comment: In this investigation you discovered that the resistance of a resistor does not change when the frequency of the AC signal applied to it changes. Ohm's law, $V = IR$ holds true at every instant in time. In Investigation 2, you will examine the behavior of capacitors and inductors with AC signals applied to them.

INVESTIGATION 2: AC SIGNALS WITH CAPACITORS

We will now explore some of these concepts. You will need the following materials:

- current probe and voltage probe
- multimeter
- 47 μF capacitor
- 800 mH inductor
- 7 alligator clip leads
- internal signal generator

Activity 2-1: Capacitors and AC Signals

In this investigation we want to see how the impedance of a capacitor changes when the frequency of the applied signal changes. You will investigate this by measuring the behavior of a capacitor when signals of various frequencies are applied to it. Specifically, you will look at the amplitudes and the relative phase of the current through it and the voltage across it.

Consider the circuit shown in Figure 9-2.

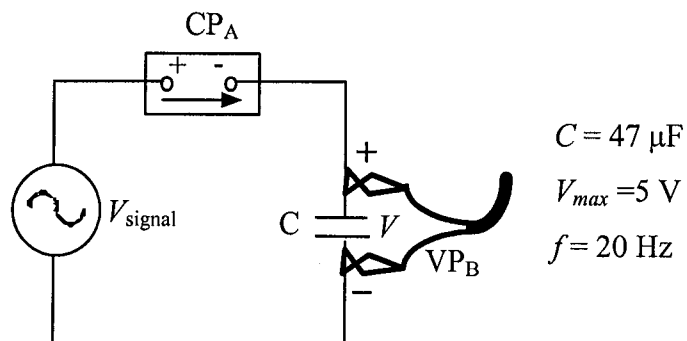
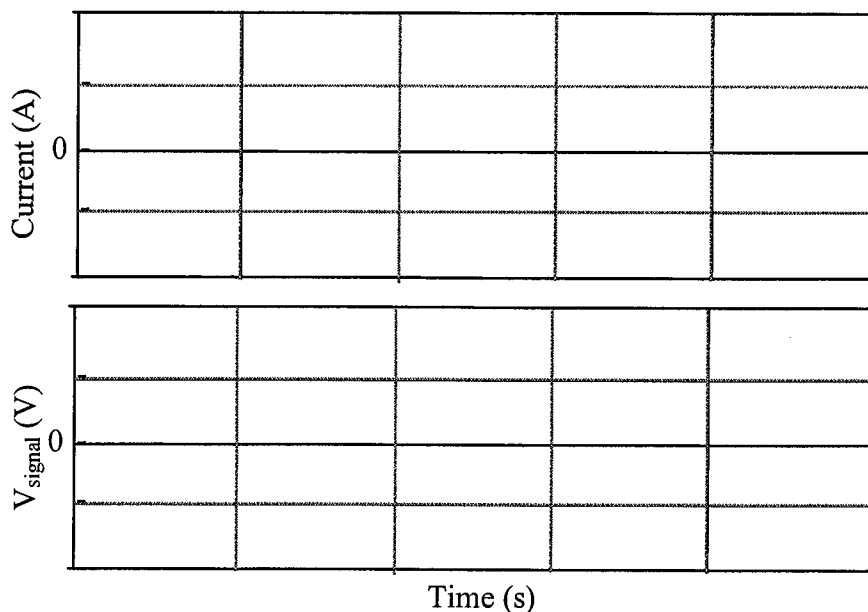


Figure 9-2: Capacitor circuit with AC input

Prediction 2-1: Suppose that you replaced the signal generator with a battery and a switch. The capacitor is initially uncharged, and therefore the voltage across the capacitor is zero. If you close the switch, which quantity reaches its maximum value first: current in the circuit or voltage across the capacitor? As charge builds up on the capacitor and the voltage across the capacitor increases, what happens to the current in the circuit? Explain.

Prediction 2-2: Sketch the AC voltage applied to the circuit in Figure 9-2 by the signal generator on the axes that follow. Use your answers from the above questions to sketch with *dashed* lines your prediction for the current as a function of time on the following graph.



Test your predictions.

1. Open the experiment file called **Capacitor**.
2. Measure the actual capacitance of C: _____
3. Connect the circuit in Figure 9-2.
4. Set the signal generator to 20 Hz and amplitude of 5 volts (+5 V maximum and -5 V minimum).
5. **Begin** graphing. When you have a good graph of the signal, **stop** graphing.
6. **Print** one set of graphs for your group report.
7. On the graph of voltage vs. time, identify and label a time or two when the current through the capacitor is maximum.
8. On your graph of current vs. time, identify and label a time or two when the voltage across the capacitor is maximum.
9. Clearly mark one period of the AC signals on your graphs.

Comment: One way you can determine the phase difference between two sinusoidal graphs with the same period is by measuring the time difference between two neighboring peaks from each graph and dividing that time difference by the period. This will give you the phase difference as a fraction of a period. For example, if the time difference between two peaks is 0.5 s and the period of the signals is 2.0 s, then the phase difference is 0.25 or $\frac{1}{4}$ period. Phase differences can also be stated in degrees or radians by simply multiplying the phase difference in periods by $360^\circ/\text{period}$ or 2π radians/period. Continuing with the example, the signals are 90° or $\pi/2$ radians out of phase. The signal that reaches its peak value first is said to *lead* the other.

Question 2-1: Does your measured voltage graph agree with your predicted one? If not, how do they differ?

Question 2-2: For the capacitor with an input signal of 20 Hz, does a current maximum occur before, after, or at the same time as the maximum voltage?

Question 2-3: What do you predict the phase difference between current and voltage (calculate)? Put your result in the table.

10. Use the various analysis features to help you fill in Column I in **Table 9-2**. Determine the experimental phase difference.

11. Set the frequency of the signal generator to 30 Hz. Check that the amplitude is still 5 V. **Graph** I and V as before. Use the **analysis feature** to complete Column II in **Table 9-2**.

Question 2-4: What can you say about the magnitude of the reactance of the capacitor at 20 Hz compared to the reactance of the capacitor at 30 Hz? Explain based on your observations.

Question 2-5: What can you say about the phase difference between current and voltage for a capacitor at 20 Hz compared to the phase difference at 30 Hz? Explain based on your observations.

Table 9-2

Column I	Column II
$f = 20\text{Hz}$	$f = 30\text{Hz}$
At maximum voltage, current is (circle one): maximum, minimum, zero and increasing, zero and decreasing, nonzero and increasing, nonzero and decreasing, other	At maximum voltage, current is (circle one): maximum, minimum, zero and increasing, zero and decreasing, nonzero and increasing, nonzero and decreasing, other
max voltage (V_{\max}) = _____	Max voltage (V_{\max}) = _____
max current (I_{\max}) = _____	Max current (I_{\max}) = _____
$Z = V_{\max}/I_{\max} =$ _____	$Z = V_{\max}/I_{\max} =$ _____
Calculate $Z \approx X_C = 1/\omega C$ _____	Calculate $Z \approx X_C = 1/\omega C$ _____
Theoretical phase diff: _____	Theoretical phase diff: _____
Experimental phase diff: _____	Experimental phase diff: _____
Current leads or voltage leads?	Current leads or voltage leads?

INVESTIGATION 3: AC SIGNALS WITH INDUCTORS

Activity 3-1: Inductors and AC Signals

In this investigation we want to see how the impedance of an inductor changes when the frequency of the applied signal changes. We will follow much the same procedure as for the capacitor.

Consider the circuit shown in Figure 9-3.

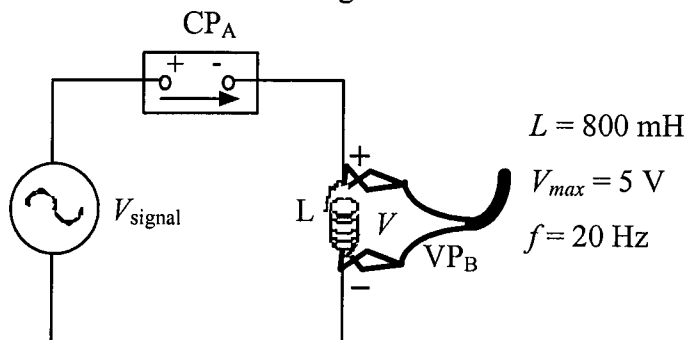
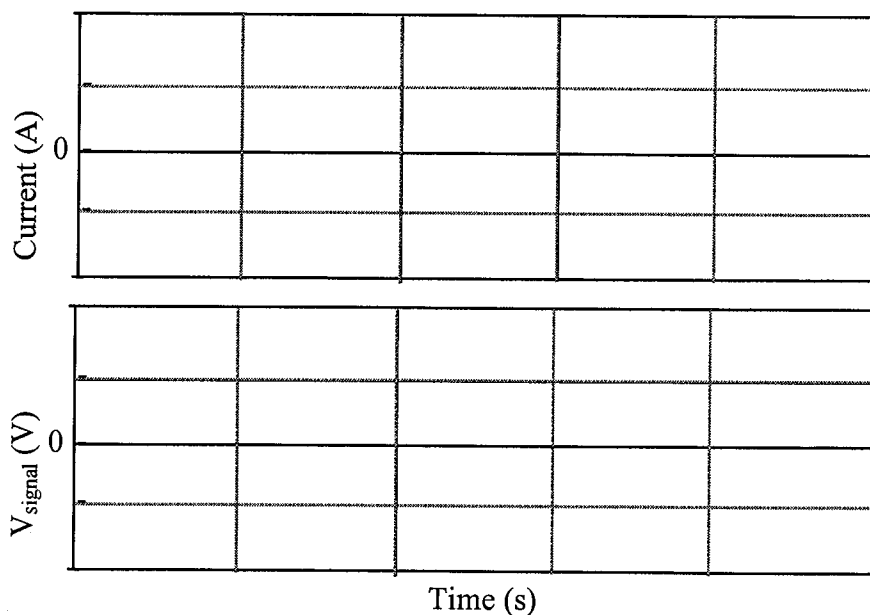


Figure 9-3: Inductor circuit with AC input

Prediction 3-1: Suppose that you replaced the signal generator with a battery and a switch. The inductor initially has no current through it. If you close the switch, which quantity reaches its maximum value first: current in the circuit or voltage across the inductor? (**Hint:** recall that when the current through an inductor is changing, the induced voltage across the inductor opposes the change.) As the current builds up in the circuit, what happens to the induced voltage across the inductor? Explain.

Prediction 3-2: At the instant the current reaches its maximum value for this circuit, what do you predict the magnitude of the induced voltage will be--maximum, minimum or zero? Why?

Prediction 3-3: Sketch the AC voltage applied to the circuit in Figure 9-3 by the signal generator on the axes that follow. Use your answers from the above questions to sketch with *dashed* lines your prediction for the current as a function of time on the following graph.



Test your predictions.

1. Open the experiment file called **Inductor**.
2. Connect the circuit in Figure 9-3.
3. Measure the actual inductance of L : _____
4. Measure the resistance R_L : _____

NOTE: The internal series resistance of the inductor is *not* negligible at these low frequencies. Hence we cannot approximate the impedance, Z , by the reactance, X , as we did in the case of the capacitor, but we must include the effect of the resistance when considering the impedance.

5. Set the signal generator to 20 Hz and amplitude of 5 volts (+5 V maximum and -5 V minimum).
6. **Begin** graphing. When you have a good graph of the signal, **stop** graphing.
7. **Print** one set of graphs for your group report.
8. On the graph of voltage vs. time, identify and label a time or two when the current through the inductor is maximum.
9. On your graph of current vs. time, identify and label a time or two when the voltage across the inductor is maximum.
10. Clearly mark one period of the AC signals on your graphs.

Question 3-1: Does your measured voltage graph agree with your predicted one? If not, how do they differ?

Question 3-2: For the inductor with an input signal of 20 Hz, does a current maximum occur before, after, or at the same time as the maximum voltage? Explain.

11. Use the analysis features to fill in Column I in **Table 9-3**. Show your calculation of the phase difference for 20 Hz in the space below the table. Determine it also for 30 Hz, but you do not need to show your work.

Question 3-3: What is the phase difference for this inductor at 30 Hz? Calculate and put in Table 9-3.

12. Set the frequency of the signal generator to 30 Hz. Check that the amplitude is still 5 V. **Graph** I and V as before. Use the analysis features to complete Column II in **Table 9-3**.

Table 9-3

Column I	Column II
$f = 20\text{Hz}$	$f = 30\text{Hz}$
At maximum voltage, current is (circle one): maximum, minimum, zero and increasing, zero and decreasing, nonzero and increasing, nonzero and decreasing, other	At maximum voltage, current is (circle one): maximum, minimum, zero and increasing, zero and decreasing, nonzero and increasing, nonzero and decreasing, other
max voltage (V_{\max}) = _____	max voltage (V_{\max}) = _____
max current (I_{\max}) = _____	max current (I_{\max}) = _____
$Z = V_{\max}/I_{\max} =$ _____	$Z = V_{\max}/I_{\max} =$ _____
Calculate $X_L = \omega L =$ _____	Calculate $X_L = \omega L =$ _____
Calculate $Z = \sqrt{R^2 + X_L^2} =$ _____	Calculate $Z = \sqrt{R^2 + X_L^2} =$ _____
Theoretical phase diff: _____	Theoretical phase diff: _____
Experimental phase diff: _____	Experimental phase diff: _____
Current leads or voltage leads?	Current leads or voltage leads?

Question 3-4: What can you say about the magnitude of the reactance of the inductor at 20 Hz compared to the reactance of the inductor at 30 Hz? Explain based on your observations.

Question 3-5: What can you say about the phase difference between current and voltage for an inductor at 20 Hz compared to the phase difference at 30 Hz? Explain based on your observations. Were the phase differences what you expected? Do you think the fact that the inductor you used has a significant resistance plays a role?

Question 3-6: Go back to **Tables 9-2** and **9-3** and compare your experimental impedances with theoretical impedances. How well do they agree? Is this reasonable?

WRAP-UP

Question 4-1: Do the reactance equations (Eq. 5) make intuitive sense for low frequencies? Answer this by considering switched DC circuits in the steady state (i.e. after things settle down). A DC current (or voltage) can be thought of as the limit of a cosine as the frequency goes to zero.

Question 4-2: Do the reactance equations (Eq. 5) make intuitive sense for high frequencies? Answer this by considering switched DC circuits immediately after the switch is closed or opened.